conventional spread measures, such as Z-spreads, are not well-defined for credit-risky bonds, especially in distressed situations.

They introduce a **survival-based valuation methodology** that accounts for the probability of default and recovery rates

This approach challenges the traditional strippable cash flow valuation assumption, emphasizing the need for models that reflect the realities of credit risk

Consistent CDS-Bond Basis

**the authors focus on the relationship** between credit default swaps (CDS) and bonds.

They propose **consistent relative value measures for CDS-bond basis trades using the bond-implied CDS term structure** derived from fitted survival rate curves

credit bonds do not have fixed cash flows. Indeed, the main (some would say the only) difference between a credit risky bond and a credit risk-free one is precisely the possibility that the issuer might default and will not honor the contractual cash flows of the bond.

Once we realize this fundamental fact, it becomes clear that the validity of the representation of a credit bond as a portfolio of cash flows critically depends on our assumption of the cash flows in case of default. In reality, once an issuer defaults it enters into an often protracted bankruptcy proceedings during which various creditors including bondholders, bank lenders,

Once an issuer defaults or declares bankruptcy its bonds trade in a fairly efficient distressed market and quickly settle at what the investors expect is the fair value of the possible recovery.

Assuming that the price of the bond immediately after default represents the fair value of the subsequent ultimate recovery cash flows, we can simply take that price as the single post-recovery cash flow which substitutes all remaining contractual cash flows of the bond

The market practice is to use the price approximately one month after the credit event to allow for a period during which investors find out the extent of the issuer’s outstanding liabilities and remaining assets

– the bond market traded more than 50 bp tighter than CDS at short maturities and more than 50 bp wider at maturities greater than 5 years, if measured on an apples-to-apples basis

Both the bonds and the CDS of Ford Motor Credit are among the most liquid

Only for the very short term exposures, where investors are forced to focus critically on the likelihood of the default scenarios and the fair value of the protection with a full account for cash flow outcomes do we see the CDS market diverging in shape from the cash market’s “optically distorted” perception of the credit spreads.

The key assumption in these models is what will be the market value of a bond just after default. In other words, one must make an assumption on what is the expected

recovery given default (or alternatively what is the loss given default). There are three main conventions regarding this assumption:

In both approaches one can find an equivalent “risky” yield curve which can be used for discounting the promised cash flows of defaultable bond

As a result, either of these approaches works quite well for creditrisky bonds that trade not too far from their par values

The main drawback of both of these approaches is that they do not correspond well to the market behaviour when bonds trade at substantial price discounts

assumption fails to recognize the fact that the market begins to discount the future recovery when the bonds are very risky. In other words, the bonds are already trading “to recovery” just prior to default,

when the expected recovery value is precisely equal to zero. We denote this as the zero recovery (ZR) assumption

Therefore, when calculating the present value of a credit bond, one can combine the riskless discount function, the survival probability, and the assumed recovery fraction into a “risky discount function” which can then be applied to the contractual cash flows

The market convention in modeling CDS spreads follows the FRP assumption and therefore the discrepancy between the CDS and conventional bond pricing models can be large in case of bonds that are trading with very wide spreads

A quantitatively consistent comparison between non-par bonds and par-equivalent CDS

To explain **"Survival-based pricing of bonds with fractional recovery of par"** from the book in a way your daughter can understand, let's turn it into a simple story with relatable concepts and visuals.

**The Story: "The Magic Bag of Coins and the Risky Borrowers"**

1. **What is a Bond?**
   * **Imagine your daughter lends her coins to her friends, and they promise to give her more coins in return after a year. This promise is called a bond.**
   * **Some friends are super reliable and always pay back (like her best friend), but others might not. Those are the risky borrowers.**

**Survival-Based Pricing (The Magic Bag of Coins)**

1. **How Lenders Think**:
   * **When your daughter lends coins, she wonders:**
     + **"Will my friend pay me back completely, partly, or not at all?"**
   * **This question is about survival—the chance that the borrower can "survive" (or stay able to repay their debt).**
2. **Fractional Recovery of Par**:
   * **Imagine one of her friends can’t pay back everything but says, "I can give you back part of what I owe, maybe half or a quarter."**
   * **This is called fractional recovery of par:**
     + **Par is the amount her friend promised to pay back.**
     + **Fractional recovery means she gets a part of it back if her friend can’t pay the full amount.**
3. **The Magic Bag**:
   * **Imagine every borrower has a magic bag of coins. The magic bag has:**
     + **A survival coin: If the borrower survives (is able to repay), they give her the full amount.**
     + **A recovery coin: If the borrower defaults (can’t repay fully), the bag still gives her a fraction of the coins.**
   * **She doesn’t know which coin will come out of the magic bag until the year ends.**

**How to Price the Bond (Deciding How Many Coins to Lend)**

1. **Thinking About Survival Chances**:
   * **Before lending, your daughter might say:**
     + **"If there’s a 90% chance my friend pays me back fully, and a 10% chance they only pay me half, how much should I charge them to borrow my coins?"**
2. **The Calculation**:
   * **To decide how many coins to lend (or what the bond is worth), she can think like this:**
     + **If the bond is worth 100 coins:**
       - **90% of the time, she gets the full 100 coins.**
       - **10% of the time, she gets only 50 coins (half of the 100 promised).**
     + **The average amount she expects to get back is: 0.9×100+0.1×50=90+5=95 coins.0.9 \times 100 + 0.1 \times 50 = 90 + 5 = 95 \text{ coins}.0.9×100+0.1×50=90+5=95 coins.**
     + **So, the bond should be priced at 95 coins, not 100, because of the risk.**

**Why This is Important**

1. **In Real Life**:
   * **Grown-ups like bankers and investors think about survival chances and fractional recovery when lending money to businesses or governments. They adjust the price of a bond based on:**
     + **How likely the borrower is to pay back.**
     + **How much they expect to recover if the borrower defaults.**
2. **Key Takeaway**:
   * **Your daughter can learn that lending always involves risks, and pricing bonds (or lending coins) means thinking about both the good (full repayment) and the bad (partial repayment).**

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Consider a credit-risky bond with maturity T that pays fixed cash flows with specified frequency (usually annual or semi-annual).

According to the fractional recovery of par assumption, the present value of such a bond is given by the expected discounted future cash flows, including the scenarios when it defaults and recovers a fraction of the face value and possibly of the accrued interest, discounted at the risk-free (base) rates

The total cash flow at each date is defined as the sum of principal ( ) i prin CF t , and interest ( ) int i CF t , payments. The integral corresponds to the recovery cashflows that result from a default event occurring in a small time interval [u,u + du]

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Estimating Survival Rates with Exponential Splines

Having derived the pricing relationship in the survival-based approach, we are now ready to estimate the implied survival probability term structure directly from bond prices.

the default hazard rate is known but the exact timing of the default event is unpredictable. T

When it comes to the estimation of term structures based on a large number of off-the-run bonds across a wide range of maturities, most approaches based on yield or spread fitting are

not adequate because they lead to a non-linear dependence of the objective function on the fit parameters. The most important aspect of this problem is the large number of securities to be fitted which makes a precise fit of all prices impractical (or not robust) and therefore creates a need for a clear estimate of the accuracy of the fit.

**The Story: "The Magic Rainbow and the Chances of Paying Back"**

1. **The Mystery of Survival Rates**:
   * **Imagine your daughter has several friends who borrow coins from her. Some friends are reliable, but others might not return all the coins.**
   * **She wants to figure out: "What are the chances my friends will be able to pay me back after 1 year, 2 years, or even 5 years?"**
   * **These chances are called survival rates. A survival rate is the likelihood that a borrower will still be able to pay back their debt at a specific time.**

**The Magic Rainbow: Smooth and Predictable Survival Rates**

1. **The Problem with Jagged Lines**:
   * **Let’s say she tries to guess the survival rates by observing her friends’ past behavior. She might plot them on a graph:**
     + **1 year: 95% chance of paying back.**
     + **2 years: 80% chance.**
     + **3 years: 75% chance.**
     + **4 years: 50% chance.**
   * **But the graph might look messy and jagged, with no clear pattern. That makes it hard to predict what will happen in between the points.**
2. **The Smooth Rainbow**:
   * **To solve this, she can use a magical math tool called exponential splines. Think of these splines as a way to draw a smooth rainbow-shaped curve through all the survival rates.**
   * **This smooth curve makes it easier to estimate survival rates for times where she doesn’t have data (e.g., "What about 2.5 years?").**

**What Are Exponential Splines?**

1. **Splines Are Like Flexible Rulers**:
   * **Imagine she’s trying to draw a smooth curve connecting all the points on her graph. If she uses a stiff ruler, the curve won’t bend nicely. But if she uses a flexible ruler (like a string), it will create a smooth, natural curve. That’s what splines do!**
   * **The exponential part just means the curve bends in a way that matches how survival rates typically behave (gradual decline over time).**

**How She Can Use the Smooth Curve**

1. **Filling in the Gaps**:
   * **Once she has her smooth rainbow curve, she can use it to figure out survival rates for any time period:**
     + **Example: If the curve says there’s a 70% chance her friend will survive (pay back) at 3.5 years, she can use that to plan her lending.**
2. **Better Decisions**:
   * **The smooth curve helps her:**
     + **Predict when her friends are most likely to struggle with repayments.**
     + **Decide how much interest to charge (higher interest for riskier friends).**
     + **Understand long-term risks better than just looking at jagged or messy data.**

**The Math Magic Behind It (Simplified)**

1. **How Splines Work**:
   * **Splines use small curves that fit together seamlessly, like puzzle pieces, to create a smooth overall curve.**
   * **The exponential part ensures that survival rates follow a natural decline over time (e.g., it gets less likely that someone will pay back the longer they borrow).**
2. **Why It’s Important**:
   * **Without splines, she’d have to guess survival rates in between her data points, and her guesses might be wrong.**
   * **Splines give her a reliable, math-backed way to make predictions.**

**Teaching Through Activities**

1. **Rainbow Curve Drawing**:
   * **Give her a piece of paper and plot some points on it (e.g., survival rates: 1 year = 95%, 2 years = 80%, 3 years = 70%, etc.).**
   * **Ask her to connect the points with a smooth curve using a string or her imagination to make a "rainbow."**
2. **Predict Survival Rates**:
   * **Create a pretend lending game where she guesses survival rates for times between the points (e.g., "What’s the survival rate at 2.5 years?").**
   * **Show her how the smooth curve helps her make better guesses compared to random guessing.**
3. **The Flexible Ruler Experiment**:
   * **Use a real ruler to draw straight lines between points (jagged prediction) and a piece of string to draw a smooth curve (spline). Let her see how much better the smooth curve looks.**

**The Key Lesson**

* **Survival rates** help us understand how likely borrowers are to pay back over time.
* **Exponential splines** are like magic rainbows that smooth out messy data, making predictions more accurate and helpful for planning.

Realizing further that the quantity which is linearly related to bond prices is the discount function, they proposed to estimate the term structure of risk-free discount function itself rather than the term structure of yields. Finally, they argued that the simplest discount function is exponentially decreasing with a constant rate

In the case of credit risky bonds a similar logic also applies, except that one has to think about the survival probabilities rather than discount function, because it is the survival probabilities that appear linearly in the bond pricing equation [11]. When the hazard rate is constant the survival probability term structure is exactly exponential. Therefore, it is indeed well suited for approximation

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**Issuer and Sector Credit Term Structures**

**The Story: "Comparing Borrowers and Their Groups"**

1. **The Kingdom of Borrowers**
   * **Imagine your daughter is the queen of a kingdom where lots of people (and even businesses!) borrow coins from her.**
   * **There are two types of borrowers:**
     + **Individuals (single borrowers, like her friends or neighbors).**
     + **Groups or sectors (like families, schools, or companies that borrow together).**
   * **She wants to figure out how risky each borrower or group is so she can decide:**
     + **"How much should I charge them?"**
     + **"How likely are they to pay me back?"**

**Issuer Credit Term Structures (Individual Borrowers)**

1. **Looking at Each Borrower’s History**
   * **For each individual borrower (like her friend Ali), she can create a timeline of their borrowing behavior:**
     + **Did they pay back their loans in the past?**
     + **How likely are they to pay back loans in the future?**
   * **This timeline is called a credit term structure. It’s a way to describe how risky a borrower is over time.**
2. **Example**:
   * **If Ali has a credit term structure that shows:**
     + **1-year survival rate: 95% chance he’ll pay back.**
     + **3-year survival rate: 80% chance.**
     + **5-year survival rate: 60% chance.**
   * **She knows Ali is more reliable for short-term loans and gets riskier for longer loans.**

**Sector Credit Term Structures (Group Borrowers)**

1. **Grouping Borrowers by Type**
   * **Now imagine her kingdom has groups of borrowers like:**
     + **Farmers (who borrow coins to grow crops).**
     + **Shopkeepers (who borrow to buy and sell goods).**
     + **Builders (who borrow to build houses).**
   * **Each group has its own sector credit term structure, showing the average risk for borrowers in that group.**
2. **Why Group Borrowers?**
   * **Instead of looking at each borrower one by one, she can group them into sectors. This helps her:**
     + **Save time by analyzing patterns for the whole group.**
     + **Compare groups to see which ones are safer or riskier.**
3. **Example**:
   * **Farmers might have a credit term structure like this:**
     + **1-year survival rate: 98% (very reliable).**
     + **5-year survival rate: 85%.**
   * **Builders might have a riskier term structure:**
     + **1-year survival rate: 90%.**
     + **5-year survival rate: 60%.**

**Comparing Issuers and Sectors**

1. **How Does She Use This Information?**
   * **By comparing individual borrowers and groups:**
     + **She can decide which ones are safest to lend to (and charge lower interest rates).**
     + **She can avoid high-risk borrowers or charge them more to cover the risk.**
2. **Example Decisions**:
   * **If a farmer asks for a 1-year loan, she might say, "I’ll charge you just a little interest because farmers are reliable."**
   * **If a builder asks for a 5-year loan, she might say, "I’ll charge you more because builders are risky over long periods."**

**Key Lessons from the Chapter**

1. **Credit Term Structures**:
   * **These are like "risk timelines" for borrowers or groups, showing how likely they are to pay back loans over time.**
2. **Issuer vs. Sector**:
   * **Issuer credit term structures look at individual borrowers.**
   * **Sector credit term structures look at groups of borrowers (like farmers, shopkeepers, or companies).**
3. **Why It Matters**:
   * **It helps lenders (like your daughter) make smarter decisions about:**
     + **Who to lend to.**
     + **How much interest to charge.**
     + **How to avoid risky borrowers or sectors.**

Thus, all of these measures neglect the dependence of the bond price on the recovery value and the debt acceleration in case of default. Therefore, these measures become inadequate for distressed bonds. In the survival-based approach, spreads are not a primary observed quantity. Only the prices of credit bonds have an unambiguous meaning. Spreads, however we define them, must be derived from the term structure of survival probabilities

It is important to remember that the implied hazard rate does not correspond to an actual forecas

Nevertheless, fitted hazard rate term structures may provide valuable clues beyond the conventional spread analysis.

For example, the Ford curve in Figure 4 is substantially wider than the sector curve, signalling the higher credit risk associated with Ford bonds

**How They All Work Together**

Here’s how these terms fit into the lending story:

1. **Par Price**: The "magic price" where the bond is traded at exactly 100 coins.
2. **Par Yield**: The fair interest rate that makes the bond worth its par price (100 coins).
3. **Par Spread**: The extra interest charged to riskier borrowers (on top of the par yield).
4. **Hazard Rate**: The borrower’s likelihood of not paying back (default risk).

**1. Par Price of the Bond: The Magic Starting Point**

* **What it is**: The **par price** of a bond is like the bond's "magic starting price," which is exactly **100 coins** (or 100% of its face value). This is the price at which the bond is issued or traded if there is no discount or premium.
* **How it works in the story**:
  + Imagine her friend Ali borrows 100 coins from her and promises to pay back **exactly 100 coins plus interest** after one year. If Ali is very reliable, she will lend him the money at this "magic price" of **100 coins**.
* **Key Point**: The **par price** assumes everything is perfect—there’s no extra risk, no late payments, and the bond pays exactly what is promised.

**2. Par Yield: The Regular Reward**

* **What it is**: The **par yield** is the interest rate (or coupon rate) on a bond that makes its price equal to its **par price** (100 coins). It’s like the "fair reward" for lending money.
* **How it works in the story**:
  + Let’s say your daughter lends 100 coins to Ali for one year. He promises to pay back the 100 coins plus **5 coins as interest**. That 5 coins is the **par yield**.
  + If Ali’s par yield is 5%, it means that lending him 100 coins today will result in **105 coins** after one year.
* **Key Point**: The **par yield** is the interest rate that keeps things balanced. If the yield goes up, the price of the bond might go down, and vice versa.

**3. Par Spread: The Risky Reward**

* **What it is**: The **par spread** is the extra reward (extra interest rate) a lender wants to charge a borrower because they are **riskier** than others.
* **How it works in the story**:
  + Imagine your daughter lends coins to two friends:
    - **Sara**: Very reliable (like lending to the government). She pays a par yield of **2%**.
    - **Omar**: A little riskier. Your daughter might charge him a higher interest rate, say **5%**.
  + The difference between Sara’s rate (2%) and Omar’s rate (5%) is the **par spread**. It’s like an extra fee for the extra risk Omar brings.
* **Key Point**: The **par spread** is the difference in yield between a risky borrower and a risk-free borrower. It tells how much more interest is needed to compensate for risk.

**4. Hazard Rate: The Risk of Not Paying Back**

* **What it is**: The **hazard rate** is the chance that a borrower will **default** (fail to pay back) during a specific time period. It’s like the "danger signal" for lenders.
* **How it works in the story**:
  + Imagine Ali has a **10% chance** of not being able to pay back the coins in a year. That 10% is his **hazard rate**.
  + If Ali’s hazard rate goes up, your daughter might decide to:
    - Charge him a higher **par spread**.
    - Stop lending him coins altogether.
* **Key Point**: The **hazard rate** is the borrower’s risk of default. Higher hazard rates mean higher risks, which lead to higher par spreads and lower bond prices.

**fitted par coupon term structure**

**What is a Coupon Rate?**:

* A **coupon rate** is just the yearly interest a borrower agrees to pay when they take out a loan. For example:
  + If Sara borrows 100 coins at a coupon rate of **5%**, she’ll pay 5 coins every year as interest.

1. **What Does "Par" Mean?**:
   * **Par means the bond (or loan) is worth exactly 100 coins. So the coupon rate is chosen in such a way that the bond starts at par value and matches the loan's payments.**

**How Does It Work?**

1. **Step 1: Collect Data**:
   * **Your daughter first collects data about interest rates in her kingdom. For example:**
     + **1-year bonds are trading at 3%.**
     + **2-year bonds are trading at 4%.**
     + **5-year bonds are trading at 5%.**
2. **Step 2: Fit the Curve**:
   * **Instead of using just the raw data, she uses a curve-fitting method (like splines or regression) to create a smooth line through the points. This smooth line shows the fitted par coupon term structure.**
3. **Step 3: Use the Curve**:
   * **Now, if someone wants to borrow for 3.5 years, she can look at the curve to find the coupon rate for that exact maturity.**

**2. Splines: Flexible Curve-Fitting**

* **What It Does**: Splines connect the data points with small, smooth pieces of curves, ensuring a flexible and seamless fit.
* **How It Works**:
  + Instead of one formula for the entire dataset, splines use **piecewise functions** (separate curves for different sections of the data).
  + The curves are connected at **knots** (data points) to ensure the transition between pieces is smooth (no sharp jumps).

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**Bond-Implied CDS (BCDS) Spread Term Structure"**

**The Story: "Borrowing Coins and Buying Insurance"**

1. **The Setup**:
   * **Imagine your daughter runs a coin-lending business. Her friends borrow coins from her, and sometimes, they might not pay her back.**
   * **To protect herself, she can buy insurance that pays her back if one of her friends defaults (doesn’t repay the loan).**
   * **This insurance is called a Credit Default Swap (CDS).**

**What is Bond-Implied CDS (BCDS)?**

1. **The Two Ways to Handle Risk**:
   * **When your daughter lends coins, she has two options:**
     1. Charge her friends a **higher interest rate** to cover the risk (this is what happens with bonds).
     2. Buy **insurance (CDS)** to protect herself in case her friends don’t pay.
2. **The Connection Between Bonds and CDS**:
   * **The Bond-Implied CDS (BCDS) is like a "price tag" that tells her how much it would cost to buy insurance (CDS) based on the extra interest (spread) her friends are already paying her for the risk.**
   * **In short: The BCDS spread shows what the cost of insurance should be, given how risky the bond (loan) is.**

**Understanding the Term Structure**

1. **What is a Term Structure?**
   * **A term structure is a timeline that shows how prices or rates change for different lengths of time.**
   * **For BCDS, the term structure shows the cost of CDS (insurance) for different loan periods:**
     + **1-year loan: How much does insurance cost?**
     + **2-year loan: How much does it cost now?**
     + **5-year loan: Is the insurance more expensive?**

**How to Calculate the BCDS Spread**

1. **Where the Numbers Come From**:
   * **The book explains that the BCDS spread is calculated by looking at two things:**
     1. **Bond Prices**: If a bond is cheap (low price), it means the borrower is risky, so CDS (insurance) will cost more.
     2. **Bond Yields/Spreads**: The higher the interest rate on a bond, the riskier it is, which means CDS prices should also go up.

1. **Example**:
   * **If Ali borrows coins at a high interest rate (10%) because he’s risky, then the cost of buying CDS insurance for his loan will also be high, maybe 8% per year.**

**Why Is This Important?**

1. **The Goal of BCDS Spreads**:
   * **BCDS spreads help lenders like your daughter figure out:**
     1. Whether it’s cheaper to charge higher interest (bond spreads) or buy insurance (CDS spreads).
     2. How much risk they’re taking on when lending coins.
   * **For example:**
     1. **If CDS insurance costs less than the extra interest she’s charging, she might want to buy insurance and keep lending at a high rate.**

**Simple Takeaways for Your Daughter**

1. **Bonds vs. Insurance**:
   * **Bonds already include the cost of risk through the extra interest (spread) charged to risky borrowers.**
   * **CDS spreads are like a separate insurance policy for lenders.**
2. **BCDS Connects Them**:
   * **The BCDS spread is the "price tag" for insurance, based on how risky the bond is.**
3. **Why Term Structures Matter**:
   * **The BCDS spread changes depending on the length of the loan (short-term vs. long-term).**
   * **Riskier borrowers and longer loans usually mean higher spreads (more expensive insurance).**

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**Bond’s fitted price" and "fitted par coupon**

**The Story: "The Fair Price and Fair Interest for Lending"**

Imagine your daughter runs a lending game where her friends borrow coins from her. She has to figure out:

1. **How much the loan (or bond) should cost today** (the **fitted price**).
2. **What interest rate to charge to make the bond perfectly fair** (the **fitted par coupon**).

**1. What is a Bond’s Fitted Price?**

* **The Problem**:
  + Your daughter notices that some of her friends’ loans are trading at odd prices. For example:
    - A loan for 100 coins might be priced at 95 coins (because the borrower is risky) or 105 coins (because the borrower is very safe).
  + She wants to figure out the **"true price"** the loan should have based on how the market sees it.
* **The Solution (Fitted Price)**:
  + To calculate the "fair price," she uses a **fitted price**. This is a price smoothed out by math tools (like regression or splines) to make sure it reflects the overall trends in the market.
* **Why It’s Useful**:
  + If one loan is priced too high or too low compared to similar loans, it might mean there’s an opportunity to lend more or avoid a risky borrower.
  + The **fitted price** gives her a better sense of what the bond should cost if there were no errors or outliers in the market.

**2. What is a Fitted Par Coupon?**

* **The Problem**:
  + She also notices that different borrowers are paying different amounts of interest (coupon rates). Some rates seem too high, and some too low, based on the borrowers' risks.
  + She wants to calculate the **perfect coupon rate** for each borrower to make their loan exactly fair.
* **The Solution (Fitted Par Coupon)**:
  + A **fitted par coupon** is the interest rate (coupon) that makes the bond worth **exactly 100 coins (par value)**.
  + It’s called "fitted" because she uses math to smooth out the coupon rates for borrowers of different risks and loan lengths (term structure).
* **Why It’s Useful**:
  + The **fitted par coupon** helps her:
    - Decide the "right" amount of interest to charge a borrower.
    - Compare borrowers and make fair deals.

**How They Work Together**

1. **Bond’s Fitted Price**:
   * **This is the fair value (price) of the loan today, based on the borrower’s risk and market trends.**
   * **If the bond's actual price is lower or higher than the fitted price, it might be an error or a special opportunity.**
2. **Fitted Par Coupon**:
   * **This is the fair interest rate (coupon) to charge the borrower to make the loan worth exactly 100 coins (par value).**
   * **If the coupon is higher or lower than the fitted par coupon, the bond might trade above or below par.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Borrower** | **Actual Price of Loan** | **Interest (Coupon)** | **Borrower’s Risk** | **Fitted Price** | **Fitted Par Coupon** |
| Ali | 95 coins | 7% | Risky | 96 coins | 6.8% |
| Sara | 105 coins | 3% | Safe | 104 coins | 3.2% |
| Omar | 100 coins | 5% | Medium Risk | 100 coins | 5% |

* The **fitted price** tells her if the loan is fairly priced:
  + Ali’s actual price (95) is close to the fitted price (96), so it’s okay.
  + Sara’s actual price (105) is slightly too high compared to the fitted price (104).
* The **fitted par coupon** tells her if the interest is fair:
  + If Ali’s coupon was 6.8% instead of 7%, the loan would be worth exactly 100 coins.
  + If Sara’s coupon was 3.2% instead of 3%, her bond would also be at par (100 coins).

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**bootstrapping method" to get the hazard curve/rate from corporate bond par yields**

**The Story: "Step-by-Step Puzzle to Find the Risk"**

Your daughter is a coin lender, and she wants to figure out **how risky her borrowers are** over time. Some borrowers might default (fail to pay back), and she wants to calculate their **hazard rates**—the chance they’ll default at each specific time, like 1 year, 2 years, or 3 years.

To do this, she needs a **hazard curve**, which shows the risk of default over time. She uses the **bootstrapping method** to build it step by step, like solving a puzzle.

**What Is the Bootstrapping Method?**

**Bootstrapping** is a step-by-step process where:

1. You start with something you already know (like par yields for bonds).
2. Use that to calculate one piece of the puzzle (hazard rates for the first year).
3. Use that first piece to calculate the next piece (hazard rates for the second year), and so on.

**How It Works for Hazard Curves**

Here’s how your daughter can use bootstrapping to calculate hazard rates from bond par yields.

**Step 1: What She Knows (Par Yields and Bonds)**

* Her friends (borrowers) issue **bonds** that promise to pay interest (coupon payments) every year and repay the full loan (par value) at the end.
* The **par yield** is the interest rate (coupon) that makes the bond’s price exactly 100 coins (par value).

**Step 2: What She Wants (Hazard Rates)**

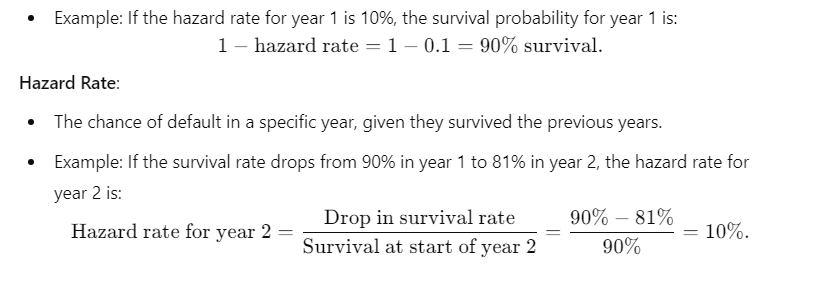
* She wants to figure out **hazard rates**, which tell her:
  + **At year 1**: What’s the chance her borrower will default?
  + **At year 2**: What’s the extra risk of default, given they survived year 1?
  + **At year 3**: And so on, for each year.

**Step 3: The Bootstrapping Process**

1. **Start with Year 1**:
   * **For the first year, she calculates the survival probability using the par yield of the 1-year bond.**
   * **Example:**
     + **Par yield = 5%.**
     + **If the borrower doesn’t default, she gets 100 coins back plus 5 coins of interest.**
     + **Using this, she calculates the hazard rate for year 1 (the risk of default in year 1).**
2. **Move to Year 2**:
   * **Now, she looks at the 2-year bond. She knows:**
     + **The borrower must survive year 1 to pay the second-year coupon and par value.**
   * **She uses the par yield of the 2-year bond and the hazard rate from year 1 to calculate the hazard rate for year 2.**
3. **Repeat for Each Year**:
   * **For a 3-year bond, she calculates:**
     + **The borrower must survive years 1 and 2 to pay the final coupon and par value.**
   * **She uses the par yield of the 3-year bond and the hazard rates for years 1 and 2 to calculate the hazard rate for year 3.**

**How Does She Calculate Survival and Hazard Rates?**

* **Survival Probability**:
  + The chance that the borrower survives (doesn’t default) until a specific year.
  + Example: If the hazard rate for year 1 is 10%, the survival probability for year 1 is: 1−hazard rate=1−0.1=90% survival.1 - \text{hazard rate} = 1 - 0.1 = 90\% \text{ survival.}1−hazard rate=1−0.1=90% survival.
* **Hazard Rate**:
  + The chance of default in a specific year, given they survived the previous years.
  + Example: If the survival rate drops from 90% in year 1 to 81% in year 2, the hazard rate for year 2 is: Hazard rate for year 2=Drop in survival rateSurvival at start of year 2=90%−81%90%=10%.\text{Hazard rate for year 2} = \frac{\text{Drop in survival rate}}{\text{Survival at start of year 2}} = \frac{90\% - 81\%}{90\%} = 10\%.Hazard rate for year 2=Survival at start of year 2Drop in survival rate​=90%90%−81%​=10%.



########## Sdummy Bonds and Risk C Calculation

She wants to figure out **how risky her borrowers are** and **how much to charge them (or how to protect herself)** when lending. She follows **3 big steps** to solve the puzzle.

**Step 1: Create Dummy Bonds**

1. **What’s a Dummy Bond?**
   * **Imagine she pretends to create a loan (bond) where:**
     + **The interest rate (coupon rate) is exactly equal to the yield—this means the loan is "fair" and priced at its "magic starting value" of 100 coins (par value).**
     + **These bonds aren’t real; they’re dummy bonds to help her do calculations.**
   * **Why? She’s trying to simplify her puzzle by starting with something "perfectly fair" that matches the market.**
2. **What Does "Priced at Par" Mean?**
   * **A bond priced at par means:**
     + **She lends 100 coins today.**
     + **She will get back exactly 100 coins (plus interest) if her borrower doesn’t default.**
   * **Example: She creates a dummy bond for Ali, who borrows coins for 1 year with a 5% coupon rate. Ali will repay 100 coins + 5 coins of interest after 1 year.**

**Step 2: Calculate the Loan’s True Value (NPV)**

1. **What is NPV?**
   * **NPV (Net Present Value) is a way to figure out how much a loan (or bond) is really worth today based on:**
     + **The money (cash flows) her borrowers promise to pay.**
     + **The risks of her borrowers defaulting.**
2. **How Does She Calculate NPV?**
   * **She uses a defaultable bond model (a way to include the chance of default). Here’s what she needs:**
     + **Cash flows: The interest and principal her borrowers promise to pay.**
     + **Treasury discount factors: These show the "safe" value of money over time.**
     + **Recovery rate: If a borrower defaults, how much money can she still get back? (e.g., 40% recovery means she gets back 40 coins for every 100 coins owed.)**
     + **Principal: The amount she lent (100 coins).**
     + **Hazard rate: The chance the borrower defaults each year.**
     + **Survival probability: The chance the borrower survives (doesn’t default) and pays back.**
3. **Example**:
   * **Ali promises to pay her 5 coins of interest and 100 coins of principal after 1 year.**
   * **If there’s a 10% chance Ali defaults, she calculates:**
     + **The expected cash flow = 90%×(105 coins)+10%×(recovery value)90\% \times (105 \text{ coins}) + 10\% \times (\text{recovery value})90%×(105 coins)+10%×(recovery value).**
     + **This gives her the "real value" of the bond.**

**Step 3: Bootstrap the Hazard Rate (Piece by Piece)**

1. **What is a Hazard Rate?**
   * **The hazard rate is the chance that her borrower will default (fail to pay) in a specific time period.**
   * **She wants to calculate the hazard rate for each year (or time period), one at a time.**
2. **Piece-Wise Hazard Rates**:
   * **Instead of guessing the hazard rate for the whole loan at once, she assumes:**
     + **The hazard rate is constant for short periods (like 1 year at a time).**
     + **Then, she solves for the hazard rate year by year (or "tenor by tenor") using her dummy bonds.**
3. **Step-by-Step Process**:
   * **For Year 1:**
     + **She uses the dummy bond and her par yield to calculate the hazard rate for the first year.**
   * **For Year 2:**
     + **She uses the Year 1 hazard rate, plus the dummy bond data for 2-year bonds, to calculate the Year 2 hazard rate.**
   * **Repeat for longer time periods to build the hazard curve.**
4. **What is the Par CDS Spread?**
   * **After calculating the hazard rate, she can figure out the par CDS spread. This is:**
     + **The cost of buying insurance to protect herself against the borrower defaulting.**
     + **It’s based on the hazard rate she just calculated.**
5. **Simple Spread Difference**:
   * **Another way she estimates risk is by comparing:**
     + **The borrower’s corporate bond par yield (the interest rate they pay).**
     + **The Treasury par yield (the interest rate on a "safe" government bond).**
   * **The difference between these two rates shows how risky her borrower is.**
   * **Example:**
     + **Ali’s par yield = 6%.**
     + **Treasury par yield = 2%.**
     + **Spread = 6%−2%=4%6\% - 2\% = 4\%6%−2%=4%.**

**Putting It All Together**

1. **Step 1**: Create **dummy bonds** with coupon rates equal to their yields. Make sure they are priced at **par** (100 coins).
2. **Step 2**: Use a **defaultable bond model** to calculate the bond’s true value (NPV), including hazard rates, survival probabilities, and recovery rates.
3. **Step 3**: Bootstrap the **hazard rate**:
   * **Start with Year 1, then calculate Year 2, and so on.**
   * **Use the hazard rates to find the par CDS spread and compare the corporate bond yield with the Treasury yield (spread difference).**

**Key Takeaways for Your Daughter**

1. **Dummy Bonds**: Pretend bonds with fair coupon rates to help calculate risk.
2. **NPV**: A way to calculate the true value of a bond by considering risk and cash flows.
3. **Bootstrap Hazard Rate**:
   * **Piece-by-piece calculation of the risk of default.**
   * **Helps build the hazard curve and figure out the cost of insurance (par CDS spread).**

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**The Story: "Building the Risk Curve for Borrowers"**

Imagine your daughter is the queen of a coin-lending kingdom. She has many borrowers (called **issuers**) who borrow coins by issuing **bonds**. Some borrowers are riskier than others, and she wants to create a **survival curve** for each borrower to figure out how likely they are to pay her back over time.

To do this, she uses an **issuer curve methodology**, which is a fancy way of saying "a system to calculate how risky each borrower is over time."

**What She’s Trying to Do**

1. **Goal**: Build a **survival curve** for each borrower. A survival curve shows the likelihood that the borrower won’t default (fail to pay) in different time periods, like 1 year, 2 years, or 5 years.
2. **How**: She uses bond market data, advanced math (like optimization), and a famous model called the **Lehman Brothers methodology**.

**Step-by-Step Process**

**Step 1: Calculate NPV Using the Defaultable Bond Model**

* **What It Means**:
  + NPV (Net Present Value) is a way to calculate how much a bond is worth today, based on:
    - The payments (cash flows) the borrower promises to make.
    - The risk that the borrower might default (hazard rate).
  + She uses a **defaultable bond model** that includes survival probabilities (the chance the borrower won’t default).
* **Example for Your Daughter**:
  + Ali borrows 100 coins for 1 year and promises to pay 5 coins of interest (coupon).
  + If Ali has a 10% chance of default, she calculates:
    - Expected payment = 90%×105 coins+10%×recovery amount90\% \times 105 \text{ coins} + 10\% \times \text{recovery amount}90%×105 coins+10%×recovery amount.
    - This gives her the NPV (real value) of the loan.

**Step 2: Model Survival Rates Using Lehman Brothers Methodology**

* **The Rule (S1 + S2 + S3 = 1)**:
  + This rule says that for each time period:
    - **S1**: The chance the borrower pays back fully (survives).
    - **S2**: The chance the borrower defaults but some money is recovered.
    - **S3**: The chance the borrower defaults and nothing is recovered.
    - These must add up to **1 (100%)**.
  + She uses this model to estimate the survival probabilities.
* **Why This Helps**:
  + By splitting risks into pieces (S1, S2, S3), she can better understand what happens if a borrower defaults.

**Step 3: Optimize the Survival Curve**

* **What She Does**:
  + She finds the best values for **S1**, **S2**, and **S3** to make sure her survival curve matches the bond market data as closely as possible.
  + She uses **optimization** to minimize errors. This means she adjusts the values until her survival curve fits the bond prices perfectly.
* **Example**:
  + If her model says Ali’s survival rate is 85% but the bond price suggests it should be 90%, she adjusts the numbers to get a better fit.

**Step 4: Compare Bond-Implied CDS Spreads**

* Once she builds the survival curve, she compares:
  + The **bond-implied CDS spread** (the cost of insurance based on her survival curve).
  + The **simple spread** (the difference between the borrower’s par yield and the Treasury yield).
* **Why This Matters**:
  + If the bond-implied CDS spread is too high or too low compared to the simple spread, it means something might be wrong with her calculations or the bond market.

**Inputs for Her Calculations**

**A. Dirty Bond Price: What Is It?**

* **Dirty Price** = The total price of the bond, including:
  + The bond’s clean price (its fair value without any extra payments).
  + Any **accrued interest** (interest that has been earned but not yet paid).
* **Example**:
  + If a borrower owes her 5 coins of interest but hasn’t paid it yet, the bond’s price will include those 5 coins as "accrued interest."

**B. Select USD Bonds with Fixed Coupons and Same Seniority**

* She only uses bonds that:
  + Are in USD (the same currency for consistency).
  + Pay fixed interest rates (to avoid complicated variable rates).
  + Have the same **seniority** (priority in repayment if the borrower defaults).
  + **Why?** This keeps her calculations simple and avoids messy data.

**C. Clean Up Callable Bonds**

* **Callable Bonds**:
  + These are bonds where the borrower can repay early (like paying off a loan before it’s due).
  + Callable bonds are unpredictable because the borrower might repay early.
  + She excludes them to keep her calculations clean and simple.

**D. Treasury Curve for Risk-Free Discount Factors**

* She uses the **Treasury curve** (interest rates for risk-free government bonds) to calculate **discount factors**:
  + These are used to figure out how much future payments are worth today (time value of money).
  + Example: Getting 100 coins in 5 years is worth less today because of inflation and risk.

**E. Transform Par Rates into Spreads**

* She converts **par rates** (interest rates for bonds) into **spreads**:
  + Spread = Borrower’s par yield - Treasury yield.
  + **Why?** This shows the extra risk of lending to her borrower compared to lending to the government.

**Simplified Example for Your Daughter**

Let’s say her borrower is Ali, and she wants to calculate his survival curve.

1. **Step 1**: She creates dummy bonds for Ali, calculates their NPV, and uses the defaultable bond model to estimate his survival probabilities.
2. **Step 2**: She models Ali’s survival rates using S1+S2+S3=1S1 + S2 + S3 = 1S1+S2+S3=1, with guesses for each time period.
3. **Step 3**: She adjusts the numbers (optimizes) to minimize the error between her model and the bond prices.
4. **Step 4**: She compares the bond-implied CDS spread to the simple spread (Ali’s par yield - Treasury yield).

**Key Takeaways for Your Daughter**

1. **Survival Curve**: A way to show how likely a borrower is to repay over time.
2. **Dirty Price**: The bond price that includes both its value and any unpaid interest.
3. **Optimization**: Adjusting numbers to make her survival curve fit the bond prices.
4. **Spread Comparison**: Looking at the difference between bond yields and Treasury yields to measure risk.

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**The Story: "Uncovering the Hidden Forces in the Lending Kingdom"**

Your daughter is still the queen of her lending kingdom, and she has already learned about calculating **hazard rates**, **spreads**, and **survival curves** for her borrowers. But now she discovers something new: **credit spreads** (the extra interest borrowers pay because of their risk) are influenced by **hidden forces**, like invisible winds blowing through the kingdom. These forces are what the **PCA/Factor-Based Methodology** tries to uncover.

**What Is PCA/Factor-Based Methodology?**

1. **The Challenge**:
   * **In her kingdom, many borrowers have credit spreads that vary for lots of reasons:**
     + **Borrower-specific risks (e.g., Ali is risky but Sara is reliable).**
     + **Economic conditions (e.g., inflation, recession, or growth).**
     + **Market-wide trends (e.g., rising interest rates).**
   * **She realizes that credit spreads aren’t random—they are driven by common factors (like hidden winds) that influence everyone.**
2. **What PCA/Factor-Based Modeling Does**:
   * **PCA (Principal Component Analysis) and factor-based modeling are like magical tools that help her:**
     + **Find the hidden forces (factors) that affect credit spreads.**
     + **Reduce complex data into a few key drivers of risk.**
   * **Instead of looking at each borrower individually (like in hazard rate models), she looks at patterns across the entire market.**

**How It Works: A Simple Breakdown**

1. **Step 1: Collect the Data**:
   * **She gathers credit spreads for many bonds in her kingdom (e.g., Ali’s bond, Sara’s bond, Omar’s bond).**
   * **Each bond’s credit spread varies over time.**
2. **Step 2: Find Patterns with PCA**:
   * **PCA is a mathematical tool that looks for patterns in the data. It helps her identify factors (hidden forces) that explain most of the variation in credit spreads.**
   * **Example:**
     + **Factor 1: A market-wide trend (e.g., an economic downturn increases spreads for everyone).**
     + **Factor 2: Industry-specific trends (e.g., farmers are riskier than shopkeepers right now).**
     + **Factor 3: Borrower-specific risks (e.g., Ali had a bad year).**
3. **Step 3: Build a Model Based on Factors**:
   * **Instead of modeling each borrower’s survival curve separately, she creates a factor-based model:**
     + **Credit Spread = Factor 1 × Weight + Factor 2 × Weight + Factor 3 × Weight + Residual.**
   * **Key Idea: The factors capture most of the variation in credit spreads, and the residual is the leftover noise.**
4. **Step 4: Apply the Model**:
   * **She uses the model to:**
     + **Predict credit spreads for new borrowers.**
     + **Understand how changes in the factors (e.g., economic growth) affect the entire kingdom.**

**How Is This Different From What We’ve Discussed So Far?**

**1. Survival Curve vs. Factor-Based Approach**

* **Survival Curve Methodology**:
  + Focuses on individual borrowers.
  + Models the chance of default (hazard rates) for each borrower over time.
  + Requires optimization for each borrower’s data.
* **Factor-Based Methodology**:
  + Focuses on the entire market.
  + Models credit spreads as being driven by a few common factors (like winds that blow across the whole kingdom).
  + Reduces complexity by summarizing many borrowers’ risks into a small number of factors.

**2. Credit Spread vs. Default Risk**

* **Survival Curve Methodology**:
  + Directly calculates the chance of default (hazard rates) and survival probabilities.
  + Focuses on individual borrower behavior.
* **Factor-Based Methodology**:
  + Models credit spreads, which include not just default risk but also market-wide influences (like liquidity, economic conditions, etc.).
  + Explains spreads rather than directly modeling default probabilities.

**3. Complexity vs. Simplicity**

* **Survival Curve**:
  + Requires detailed bond data for each borrower.
  + Is more precise for individual borrower risk.
* **Factor-Based**:
  + Uses broad market data to find patterns.
  + Is simpler for understanding overall market trends.

**A Fun Analogy: Comparing the Two Methods**

1. **Survival Curve**:
   * **Imagine she’s studying one borrower at a time.**
   * **She builds a detailed profile of each borrower’s risk (e.g., Ali’s hazard rate and survival curve).**
2. **Factor-Based Methodology**:
   * **Imagine she’s studying the entire kingdom.**
   * **She notices that all borrowers seem to be affected by hidden forces, like the wind, and uses PCA to uncover those forces (factors).**

**Key Terms for Your Daughter**

1. **Credit Spread**:
   * **The extra interest borrowers pay because of their risk.**
2. **Factors**:
   * **Hidden forces that influence credit spreads for many borrowers at once (e.g., market trends, economic conditions).**
3. **PCA (Principal Component Analysis)**:
   * **A mathematical tool that finds patterns and reduces complex data into a few key factors.**

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To explain **PCA (Principal Component Analysis)** to your daughter, let’s break it into a super simple, hands-on story that shows her how PCA works step by step. We'll focus on finding **patterns** in data and reducing complexity in a way that feels fun and easy.

She wants to **find the most important patterns** in this data and **simplify** it without losing the key information. That’s where **PCA (Principal Component Analysis)** comes in. PCA is like a magical tool that helps her uncover hidden patterns and organize the data better.

**What PCA Does (In Simple Terms)**

1. **Find the Patterns**:
   * **PCA looks at messy data and figures out the most important directions or patterns that explain the biggest changes in the data.**
2. **Reduce Complexity**:
   * **Instead of tracking every single detail, PCA reduces the data to just a few main patterns (called principal components) that explain most of what’s going on.**
3. **Simplify Decisions**:
   * **Once she knows the main patterns, she can use those to understand her borrowers and make better lending decisions.**

**How PCA Works: Step-by-Step**

**Step 1: Gather Data**

* She collects information about her borrowers. For example:
  + Borrower Ali: Credit spread changes every year.
  + Borrower Sara: Spread changes differently every year.
  + Borrower Omar: Similar patterns but slightly different.

**Example Table of Data:**

* Rows = Borrowers (Ali, Sara, Omar)
* Columns = Spreads in each year (Year 1, Year 2, Year 3, etc.)

|  |  |  |  |
| --- | --- | --- | --- |
| **Borrower** | **Year 1 Spread** | **Year 2 Spread** | **Year 3 Spread** |
| Ali | 5% | 6% | 4% |
| Sara | 3% | 4% | 2% |
| Omar | 4% | 5% | 3% |

* **Challenge**: The data looks messy. It’s hard to see patterns just by staring at it.

**Step 2: Center the Data**

* PCA starts by "centering" the data:
  + Subtract the average value for each column (e.g., for Year 1, subtract the average spread from Ali, Sara, and Omar’s Year 1 spreads).
  + This step focuses on how each borrower’s spread **deviates** from the average.

**Step 3: Find the Hidden Patterns**

* Now PCA works its magic:
  1. **Look for Directions of Variation**:
     + **PCA finds the direction where the data varies the most (this is the first principal component).**
     + **Think of it like drawing a line through the data points where they are most spread out.**
  2. **Add More Components**:
     + **Once it finds the first direction, PCA looks for the next most important direction (the second principal component) that’s at a right angle to the first.**
     + **This continues until all important patterns are captured.**

**What Are Principal Components?**

* **Principal components** are the "directions" or patterns in the data that explain the most variation.
* **Example**:
  + Principal Component 1 (PC1): "Borrowers’ spreads tend to move up or down together based on the market."
  + Principal Component 2 (PC2): "Some borrowers are riskier than others, even when the market is stable."

**Step 4: Simplify the Data**

* After finding the principal components, PCA can:
  + Transform the messy data into just a few numbers (the borrower’s score on each principal component).
  + For example:
    - Instead of tracking 3 years of spreads for each borrower, PCA summarizes Ali as:
      * PC1 Score = 2.5
      * PC2 Score = 1.0
  + This reduces the complexity while keeping most of the important information.

**Why PCA Is Useful**

1. **Reduces Complexity**:
   * **Instead of tracking all the data, PCA focuses on just a few patterns.**
   * **For example, instead of 3 years of spreads, PCA might summarize everything into 2 key principal components.**
2. **Finds Hidden Patterns**:
   * **PCA uncovers trends she might not have noticed, like:**
     + **"Borrowers’ spreads tend to move together when the market changes."**
     + **"Some borrowers behave differently than the rest."**
3. **Makes Predictions Easier**:
   * **With fewer patterns to focus on, it’s easier to understand how borrowers’ spreads will change in the future.**